Active & Interactive Graph Matching

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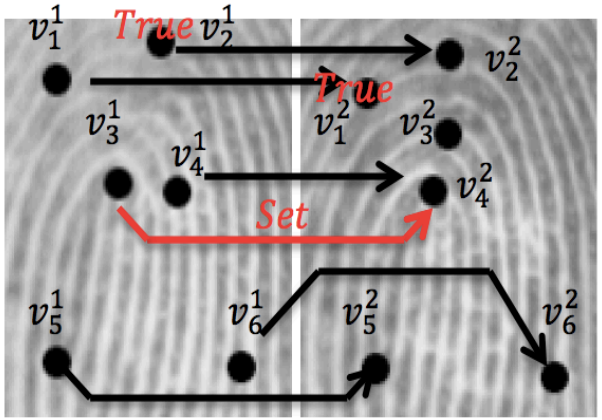

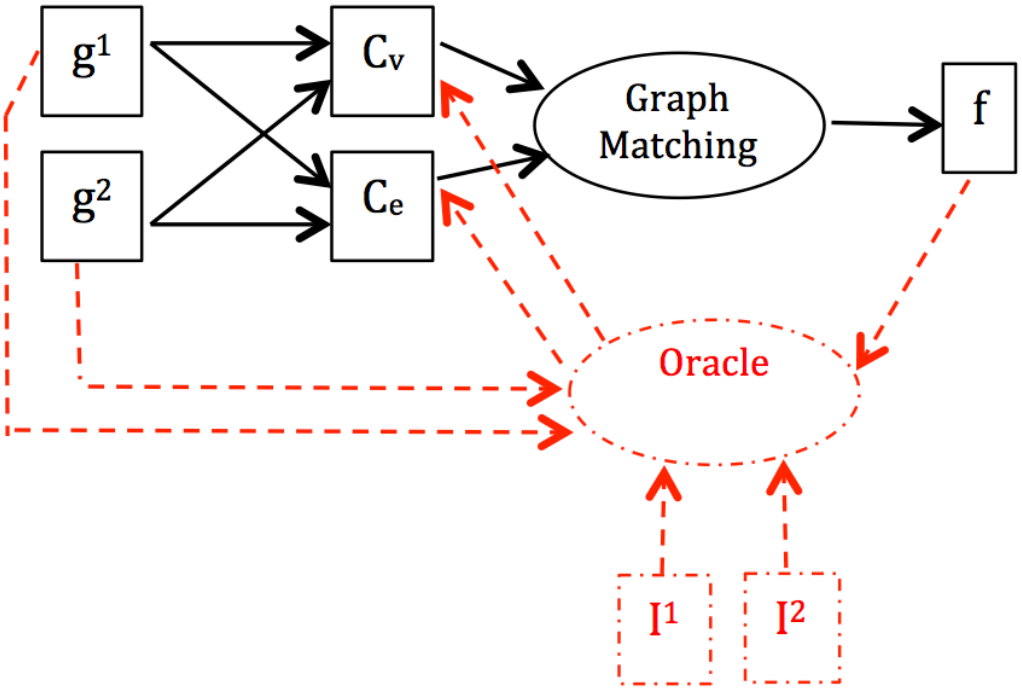

Active & Interactive Graph Matching

Graph matching: The aim is to find the best labelling between nodes of two graphs such that the cost of this labelling is the minimum among all possible labellings.

Active learning: The aim is to achieve a greater accuracy with fewer classified training examples through choosing the data from which it learns.

Interactive learning: The aim is to query some selected data and present it to an oracle (automatic system or a human annotator) for correctly classify it.

Our model:

- The learner queries the graph node that it is supposed to produce a greater impact on the labelling between both graphs.

- The oracle answers which is the graph node of the second graph that it has to be matched.

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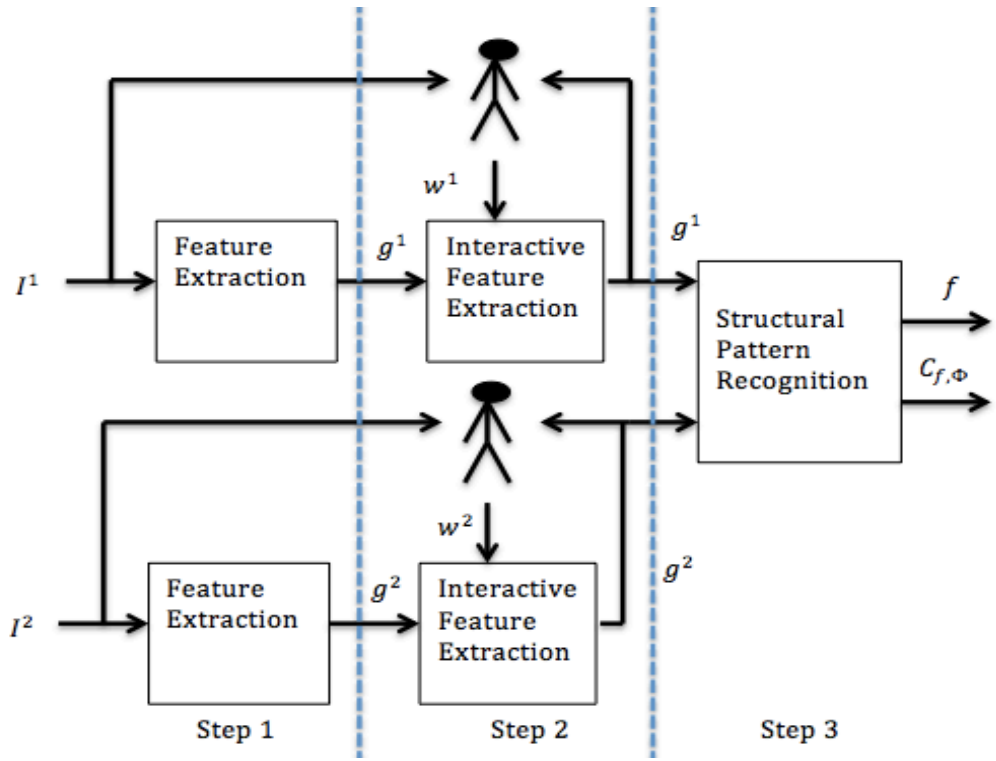
Image Correspondence Process with human interaction



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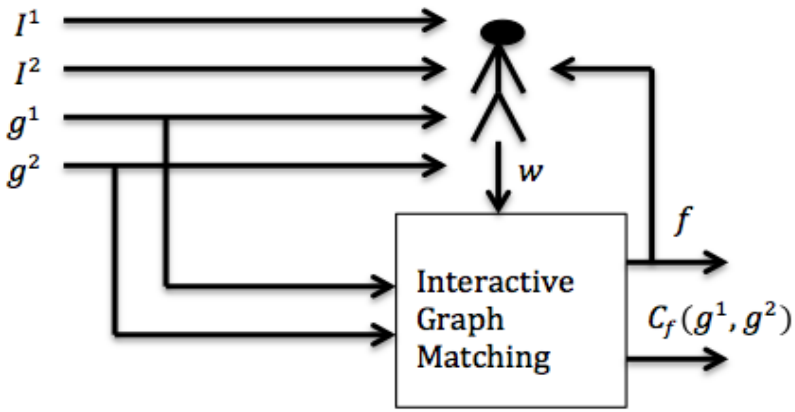
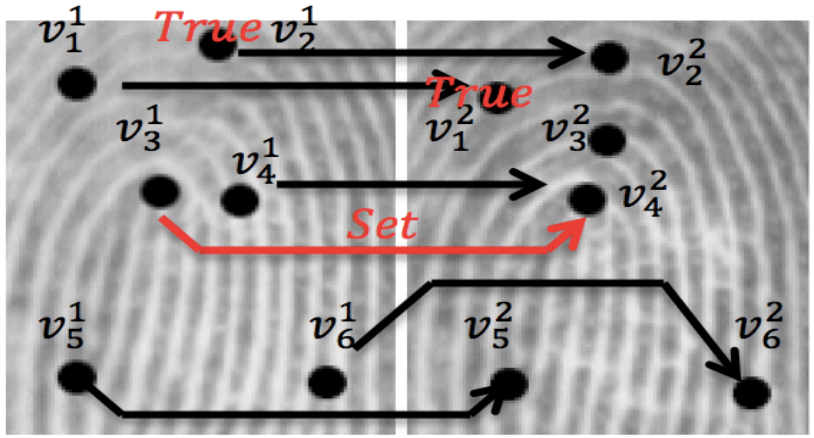
Interactive Graph Matching





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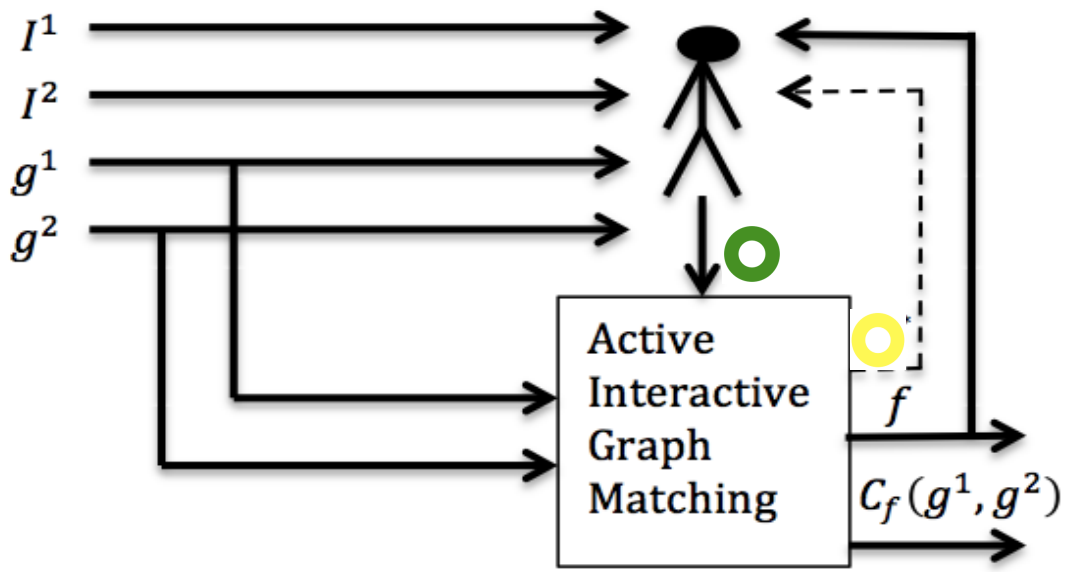
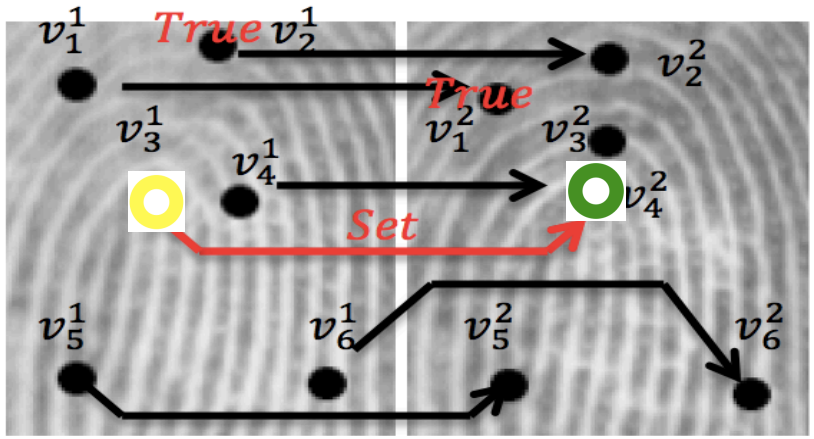
Active & Interactive Graph Matching





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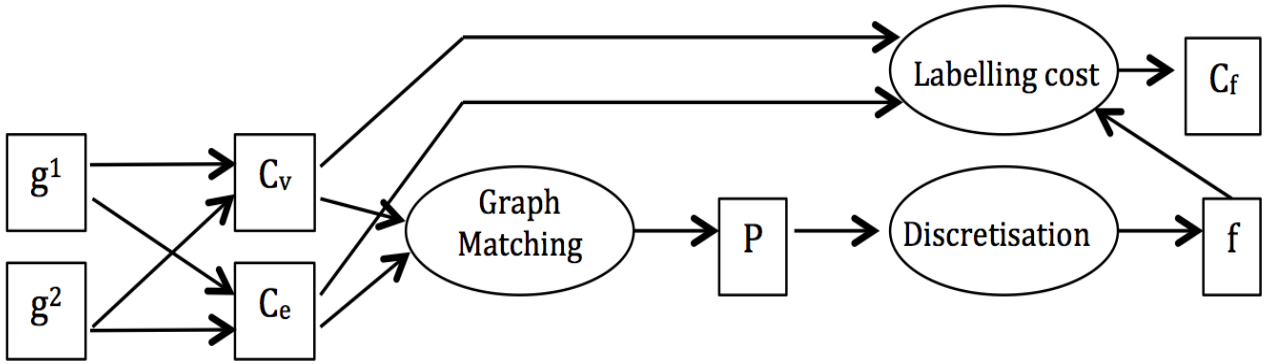
Structural Pattern Recognition based on Graphs

$$C_f(g^1, g^2) = \sum_{v_i^1 \in \Sigma_v^1} c_v(v_i^1, v_a^2) + \sum_{e_{ij}^1 \in \Sigma_e^1} c_e(e_{ij}^1, e_{ab}^2)$$



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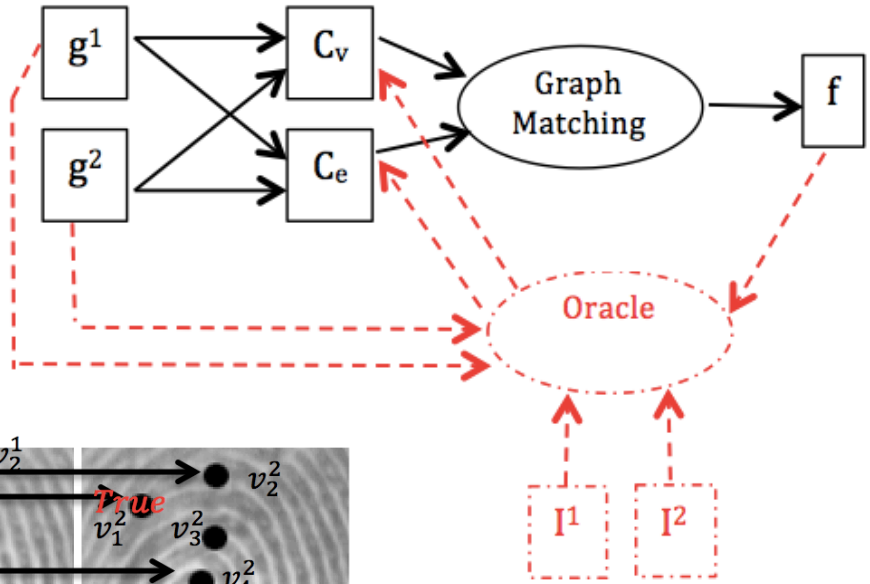
Interactive Graph Matching



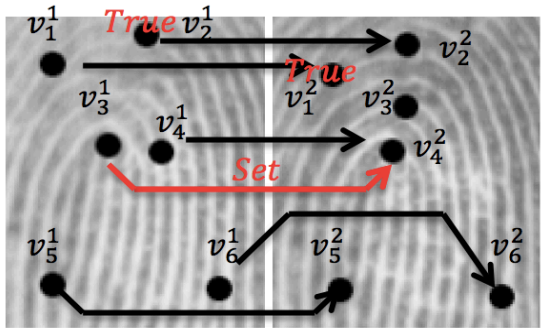
Node Costs:

If $True(v_i^1, v_a^2)$ **then**

$$C_v[i,a] = 0$$
$$C_v[i,b] = \infty \ \forall b \neq a$$
$$C_v[j,a] = \infty \ \forall j \neq i$$

If $False(v_i^1, v_a^2)$ **then**

$$C_v[i,a] = \infty$$



Arc Costs:

If $True(v_i^1, v_a^2) \wedge True(v_j^1, v_b^2)$ **then**

$$C_e[i,j,a,b] = 0 \wedge C_e[j,i,b,a] = 0$$
$$C_e[i,j,a',b'] = \infty \wedge C_e[j,i,a',b'] = \infty \ \ \forall a \neq a' \text{ and } b \neq b'$$
$$C_e[i',j',a,b] = \infty \wedge C_e[i',j',b,a] = \infty \ \ \forall i \neq i' \text{ and } j \neq j'$$

If $False(v_i^1, v_a^2)$ **then**

$$C_e[i,j',a,b'] = \infty \wedge C_e[j',i,b',a] = \infty \wedge C_e[j',i,a,b'] = \infty \wedge C_e[j',i,a,b'] = \infty \ \forall i \neq j' \text{ and } a \neq b'$$

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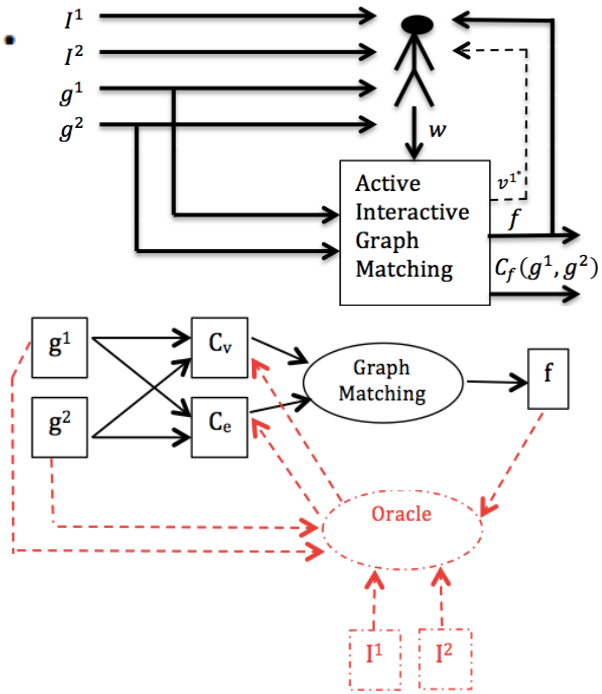
Active & Interactive Graph Matching

Algorithm *Active Graph Matching*
Input: Attributed Graphs g^1 and g^2
Output: Labelling f and Cost C_f
$C_v^0, C_e^0 = Initialise\_Cost(g^1, g^2)$; $C_v = C_v^0$; $C_e = C_e^0$.
$f = Graph\_Matching(C_v, C_e)$.
Do
$v^{1^*} = Active\_Query(P, f)$.
$v^{2^*} = Oracle\_Feedback(g^1, g^2, v^{1^*}, f)$.
$w_1 = Set(v^{1^*}, v^{2^*})$.
$C_v = Interactive\_Node\_Costs(w, C_v)$.
$C_e = Interactive\_Edge\_Costs(w, C_e)$.
$f = Graph\_Matching(C_v, C_e)$.
Since *Stop*
Compute $C_f(C_v^0, C_e^0)$
End Algorithm



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Active Learning Strategies

Four strategies to select a node v^{1*} of g^1 that have to be queried to an oracle.

The oracle feedback is the node of g^2: $v^{2*} = f(v^{1*})$

Least Confident (LC)

Least Confident given the Current Labelling (LCCL)

Maximum Entropy (ME)

Expected Model Change (EMC)

Least Confident (LC)

This strategy queries the node that its highest probability of belonging to a class is the lower one between all the elements.

$$v^{2^{(i)}} = \underset{\forall j=\{1,..,n\}}{\operatorname{argmax}} P\left[v_i^1, v_j^2\right];\ \forall i = \{1,..,n\}$$

$$v_{LC}^{1^*} = \underset{\forall i=\{1,..,n\}|Q(i)=False}{\operatorname{argmin}} P\left[v_i^1, v^{2^{(i)}}\right]$$

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Least Confident given the Current Labelling (LCCL)

This strategy queries the node that has the minimum probability given the current labelling.

$$v_{LCCL}^{1^*} = \underset{\forall i=\{1,..,n\}|Q(i)=False}{\operatorname{argmin}} P[v_i^1, f(v_i^1)]$$

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Maximum Entropy (ME)

This strategy queries the node that has the maximum Shanon Entropy given the probabilities.

$$v_{ME}^{1^*} = \underset{\forall i=\{1,..,n\}|Q(i)=False}{\operatorname{argmax}} -\sum_{j=1}^{n} P[v_i^1, v_j^2] \cdot log(P[v_i^1, v_j^2])$$

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Expected Model Change (EMC)

This strategy queries the node that would impart the greatest change to the current labelling if we knew its class.

$$R_i = \max_{\forall j=\{1,..,n\}} \{P[v_i^1, v_j^2]\} - P[v_i^1, f(v_i^1)]$$

$$v_{EMC}^{1^*} = \underset{\forall i=\{1,..,n\} \wedge Q(i)=False}{\operatorname{argmax}} \{R_i\}$$

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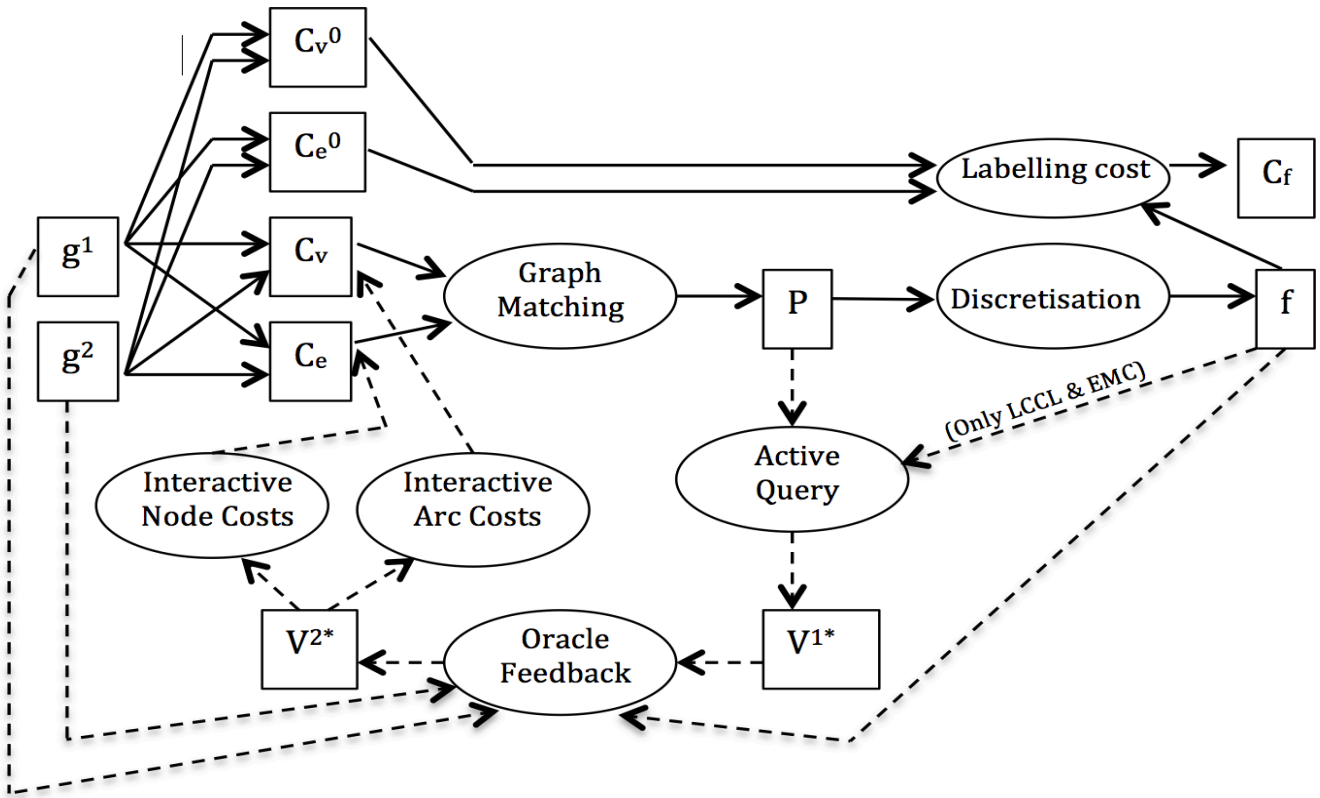

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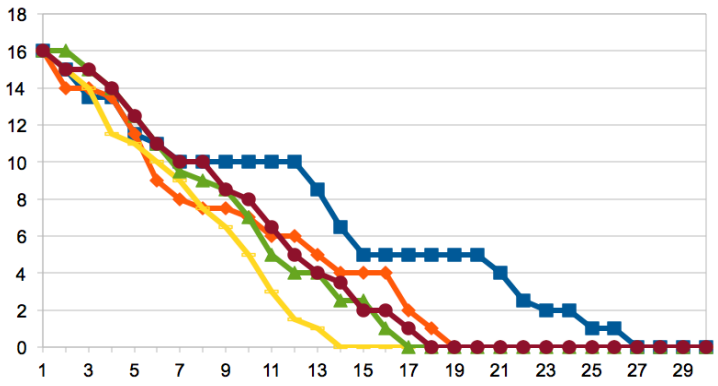
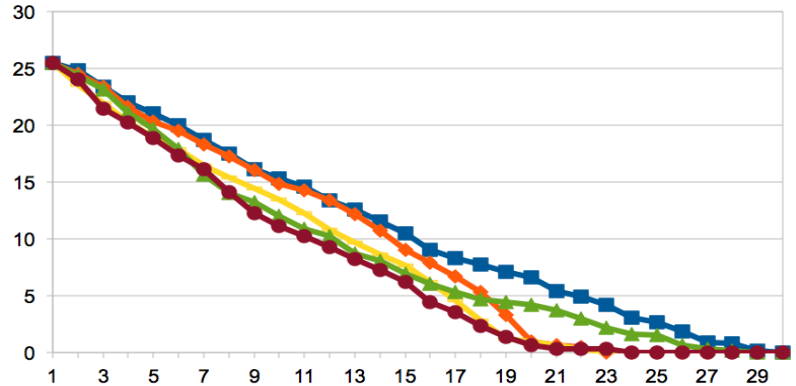
Practical Evaluation

Hamming distance respect of the number of iterations



Hotel



House

LCCL: , LC: , ME: , EMC: and Random:

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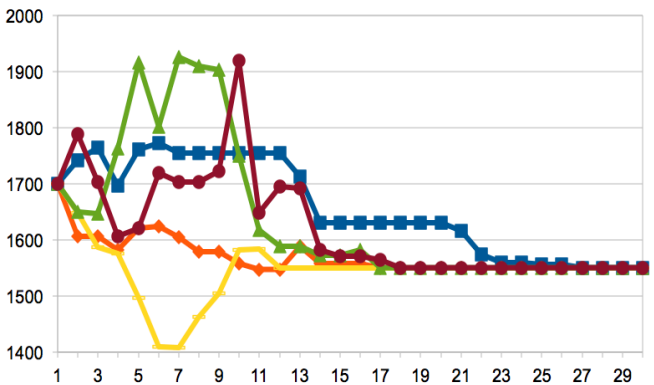
Practical Evaluation

Matching cost respect of the number of iterations



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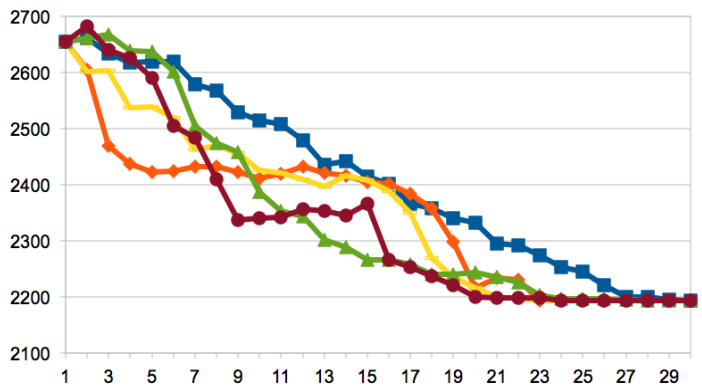

House

LCCL: , LC: , ME: , EMC: and Random:

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Conclusions

-Four different strategies to be applied on an active graph-matching algorithm

-It is not needed to modify the code of the graph matching algorithms:

- They read the probability matrix and
- write the matrix costs

-Experimental validation shows that the Least Confident with Current labelling (LCCL) tends faster to find the optimal labelling