Product graph based inexact subgraph matching and its application in symbol spotting in graphical documents

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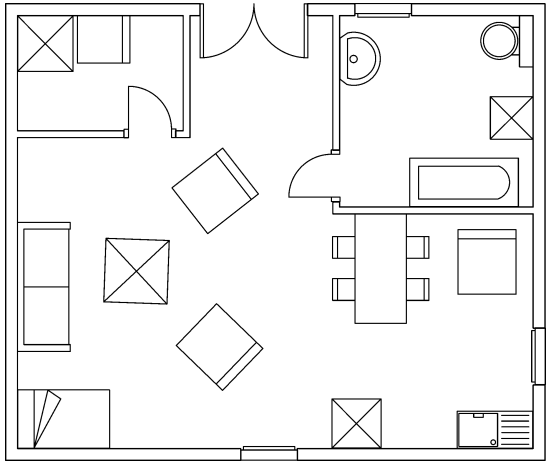
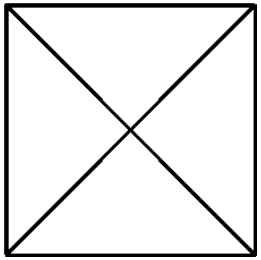
February 21, 2014

Symbol spotting

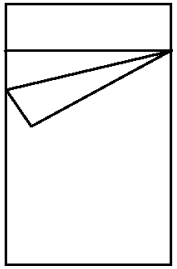
- Symbol spotting can roughly be defined as locating a given query symbol in a large graphical document or a collection of graphical document.
- The problem of symbol spotting in graphical documents can trivially be formulated as a subgraph matching problem.



(a) graphical document



(b) table1



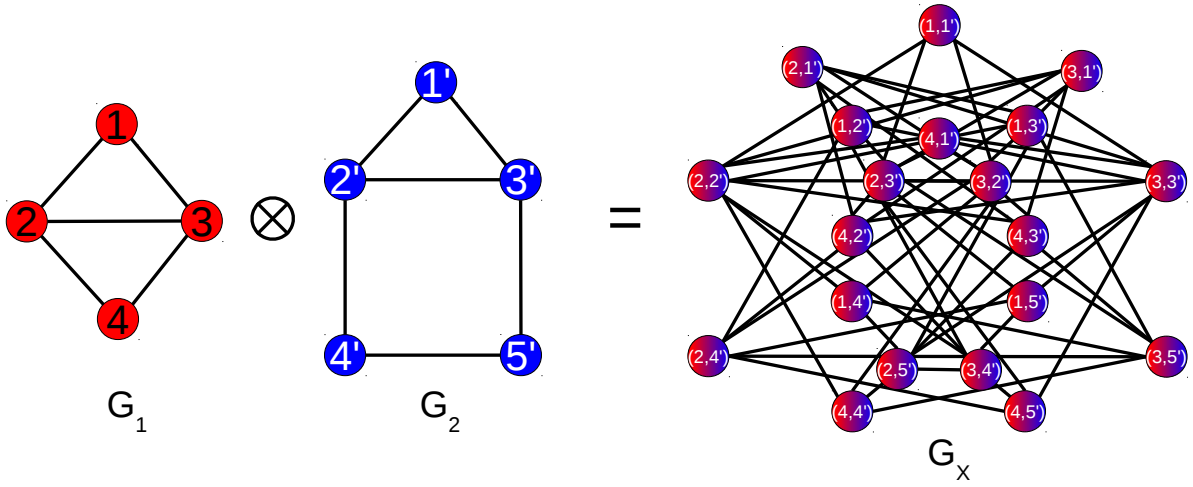
(c) bed

Subgraph matching

- Subgraph matching can trivially be formulated as an integer quadratic programming (IQP) which is an NP-hard problem.
- Many of the available subgraph matching algorithms focus on approximating the matching part like approximate maximal clique finding, linear programming, continuous relaxation etc.
- Most of them work with pairwise similarities, which are not robust in many cases especially when context informations are involved like graph.
- Incorporation of contextual information in similarity measurement adds more discrimination and able to better approximate the matching part. This is the main motivation of the work.

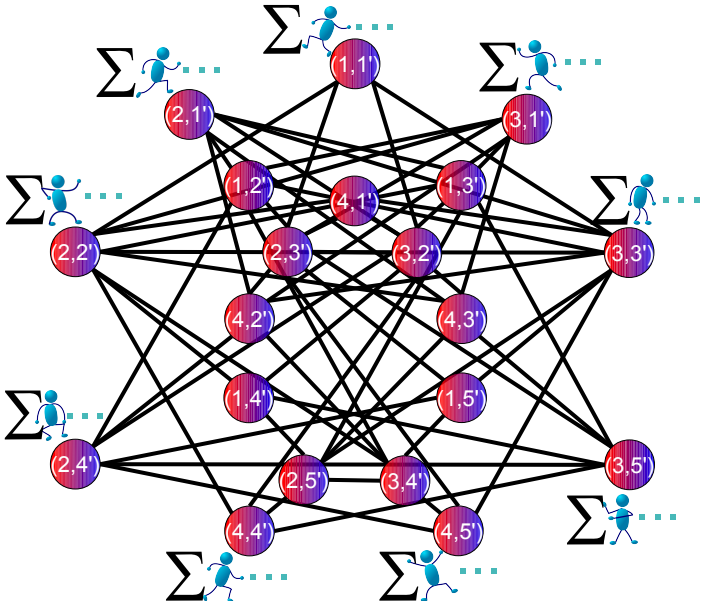
Higher order contextual similarities

- Obtaining contextual information with the walk based propagation of pairwise similarities in tensor product graph (TPG).
- A TPG is a triplet $G_X = (V_X, E_X, W_X)$ where one can assign pairwise similarities as weights on the nodes and the edges.



Higher order contextual similarities (contd.)

- Given the pairwise similarities, one can perform random walks from node to node considering those weights as the plausibilities.
- Add the weighted walks originated from a particular node.
- Higher order contextual similarity implies higher chance of constituting a pair of matched nodes.



Higher order contextual similarities (contd.)

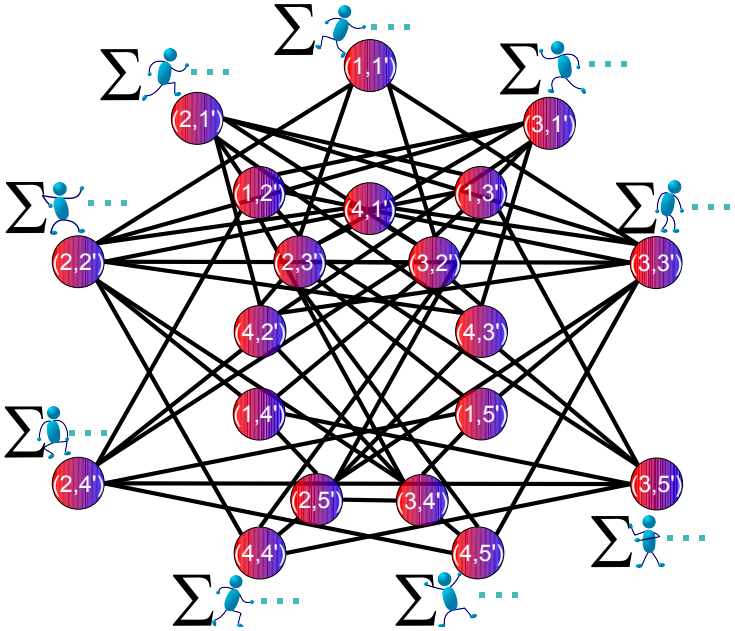
- The contextual similarities can be obtained by simple algebraic operation as follows:
- Random walks:

$$W_X^{CS} = (\mathbf{I} - \lambda W_X)^{-1}\mathbf{1}$$

- Backtrackless walks:

$$W_X^{CS} = (1-\lambda^2)(\mathbf{I}-\lambda W_X+\lambda^2 Q_X)^{-1}\mathbf{1}$$

where $Q_X = \text{diag}(\text{diag}(W_X^2)-1)$

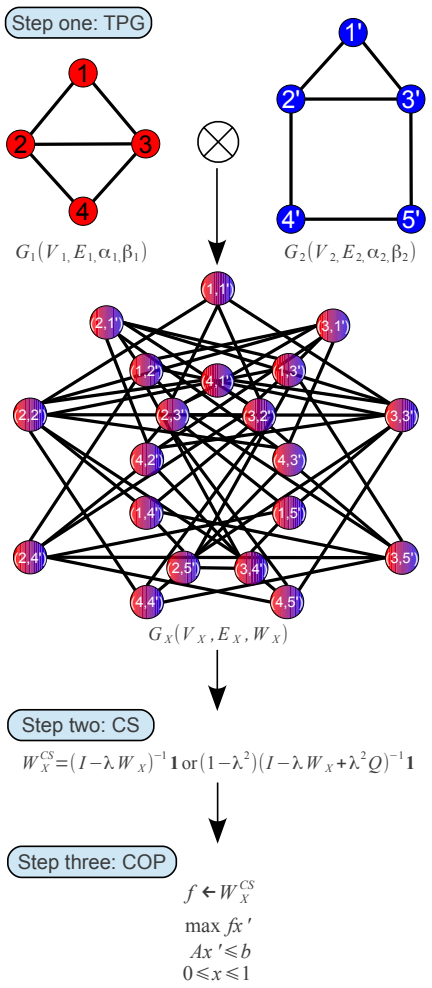


Constrained optimization problem

- We formulate subgraph matching as a node, edge selection problem from the TPG.
- We construct a constrained optimization problem out of the contextual similarity values:

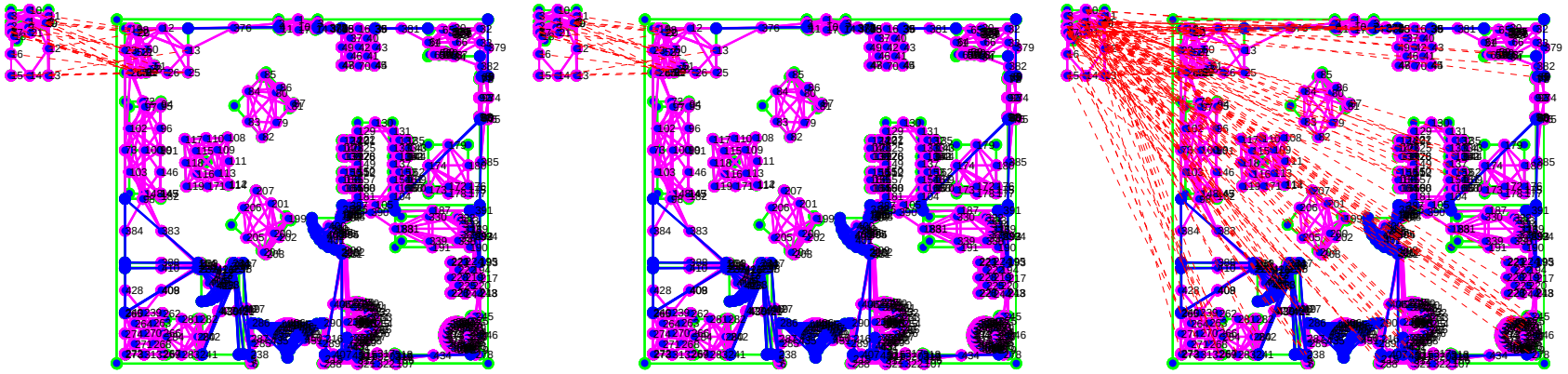
$$\begin{aligned} \max\ & fx' \\ & Ax \leq b \\ & x \in [0,1] \end{aligned}$$

Outline


Step one: TPG
$G_1(V_1, E_1, \alpha_1, \beta_1)$
$G_2(V_2, E_2, \alpha_2, \beta_2)$
$G_X(V_X, E_X, W_X)$
Step two: CS
$W_X^{CS} = (I - \lambda W_X)^{-1}\mathbf{1}$ or $(1-\lambda^2)(I - \lambda W_X + \lambda^2 Q)^{-1}\mathbf{1}$
Step three: COP
$f \leftarrow W_X^{CS}$
$\max fx'$
$Ax' \leqslant b$
$0 \leqslant x \leqslant 1$

Experimental results

Inexact graph matching (symbol spotting).



(d) random **(e)** backtrackless **(f)** pairwise similarities

Figure: Matchings: *bed*.

Experimental results (contd.)

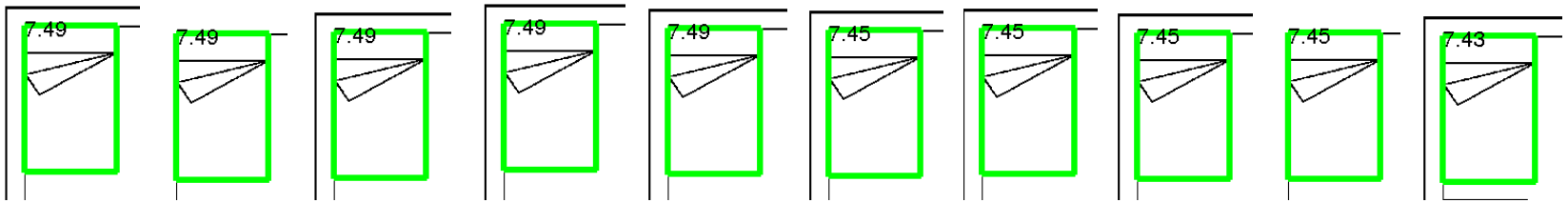


Figure: Ranked results: *bed*.

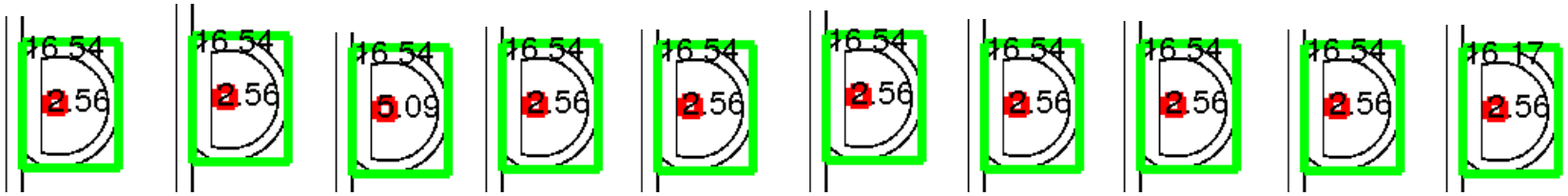


Figure: Ranked results: *door1*.

Conclusions

- Contextual similarities instead of pairwise similarities, contextual similarities encode higher order intrinsic relations between objects. Better discriminations.
- Subgraph matching as a node, edge selection problem in TPG, constrained optimization problem.
- Reason to visit the poster: details of the methodologies, results, discussions etc. Exchange ideas, feed back etc.

Dual graph